

Problem Set 4

$$\textcircled{1} \quad E(r_p) = r_f + \beta_p (E(r_M) - r_f)$$

$$\beta_p = \frac{0.18 - 0.06}{0.14 - 0.06} = 1.5$$

$\textcircled{2} \quad a.$

$$\sigma_A = \sqrt{0.8^2 \times 0.22^2 + 0.3^2} = 34.78\%$$

$$\sigma_B = \sqrt{1.2^2 \times 0.22^2 + 0.4^2} = 47.93\%$$

b.

$$\begin{aligned} \mu_p &= 0.3 \times 0.13 + 0.45 \times 0.18 + 0.25 \times 0.08 \\ &= 14\% \end{aligned}$$

$$\sigma_{AB} = 0.8 \times 1.2 \times 0.22^2 = 0.046464$$

$$\begin{aligned} \sigma_p &= \sqrt{0.3^2 \times 0.3478^2 + 0.45^2 \times 0.4793^2} \\ &\quad + 2 \times 0.3 \times 0.45 \times 0.046464 \\ &= 26.45\% \end{aligned}$$

$$\begin{aligned} \beta_p &= 0.3 \times 1.2 + 0.45 \times 0.8 + 0.25 \times 0 \\ &= 0.78 \end{aligned}$$

$$\sigma(e_p) = \sqrt{0.2645^2 - 0.78^2 \times 0.22^2} = 20.12\%$$

- ③ a. Stock A has more firm-specific risk since $\sigma(e_A) = 10.3\% > 9.1\% = \sigma(e_B)$.
- b. Stock A has more market risk since $\beta_A = 1.2 > 0.8 = \beta_B$.
- c. For stock A since $R\text{-square}(A) = 0.576 > 0.436 = R\text{-sq}(B)$.
- d. $r_A - r_F = 1\% + 1.2(r_M - r_F) + e_A$
- $$r_A = \underbrace{1\% + (1 - 1.2)r_F}_{\text{new intercept} = -0.2\%} + 1.2r_M + e_A$$

④ a. $R\text{-square} = \frac{\beta^2 \sigma_M^2}{\sigma^2}$

$$\Rightarrow \sigma = \sqrt{\frac{\beta^2 \sigma_M^2}{R\text{-square}}}$$

$$\sigma_A = \sqrt{\frac{0.7^2 \times 0.2^2}{0.2}} = 31.30\%$$

$$\sigma_B = \sqrt{\frac{1.2^2 \times 0.2^2}{1.2}} = 69.28\%$$

$$b. \sigma_A^2 = 0.098 = \underbrace{0.0196}_{20\% \text{ systematic}} + \underbrace{0.0786}_{\text{the rest.}}$$

$$\sigma_B^2 = 0.48 = \underbrace{0.0576}_{12\% \text{ systematic}} + \underbrace{0.4224}_{\text{the rest.}}$$

$$c. \sigma_{AB} = \beta_A \beta_B \sigma_H^2 = 0.7 \times 1.2 \times 0.2^2 = 0.0336$$

$$\rho_{AB} = \frac{0.0336}{0.3130 \times 0.6928} = 0.155$$

$$d. \sigma_{AH} = 0.7 \times 1 \times 0.2^2 = 0.028$$

$$\sigma_{BH} = 1.2 \times 1 \times 0.2^2 = 0.048$$

e.

$$\sigma_P = \sqrt{0.6^2 \times 0.098 + 0.4^2 \times 0.48 + 2 \times 0.6 \times 0.4 \times 0.0336} = 35.81\%$$

$$\beta_P = 0.6 \times 0.7 + 0.4 \times 1.2 = 0.9$$

$$\sigma^2(e_P) = \sigma_P^2 - \beta_P^2 \sigma_H^2$$

$$= 0.3581^2 - 0.9^2 \times 0.2^2$$

$$= 0.095808$$

$$\sigma_{PH} = 0.9 \times 1 \times 0.2^2 = 0.036$$

f.

$$\sigma_p = \sqrt{0.5^2 \times 0.3581^2 + 0.3^2 \times 0.2^2 + 2 \times 0.5 \times 0.3 \times 0.036}$$
$$= 29.48\%$$

$$\beta_Q = 0.5 \times 0.9 + 0.3 \times 1 + 0.2 \times 0 = 0.75$$

$$\sigma^2(e_Q) = 0.2948^2 - 0.75^2 \times 0.2^2 = 0.064404$$

$$\sigma_{QM} = 0.75 \times 1 \times 0.2^2 = 0.03.$$

(5)

$$a. \quad SD = \frac{D}{0.14} \Rightarrow D = SD \times 0.14 = 7.$$

$$b. \quad 0.14 = 0.06 + \beta \times 0.085$$

$$\beta_{old} = \frac{0.08}{0.085} = 0.941$$

$\beta = \rho \frac{\sigma}{\sigma_M} \Rightarrow$ if ρ doubles then β also doubles.

$$\beta_{new} = 2 \times 0.941 = 1.882$$

$$E(\sigma_{new}) = 0.06 + 1.882 \times 0.085 = 22\%$$

$$P_{new} = \frac{7}{0.22} = \$31.82$$

⑥

a. Only beta matters here.

$$E(r_{\$1}) = 0.04 + 1.5 \times 0.06 = 13\%$$

$$E(r_{\$5}) = 0.04 + 1 \times 0.06 = 10\%$$

b. According to the CAPM, \$1 store should yield more than expected. The stock is overpriced.

Also, CAPM says that the return on everything \$5 should be less than expected. The stock is undervalued.

⑦

$$0.12 = r_F + 1.2 (E(r_M) - r_F)$$

$$0.09 = r_F + 0.8 (E(r_M) - r_F)$$

$$\Rightarrow 0.03 = 0.4 (E(r_M) - r_F)$$

$$\Rightarrow 0.12 = r_F + 1.2 \times \frac{0.03}{0.4}$$

$$r_F = 0.12 - 0.09 = 3\%$$